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Grassmann manifold Bosonization of QCD in Two Dimensions

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ABSTRACT

Two dimensional QCD is bosonized to be an integrably deformed Wess-Zumino-Witten model under proper limit. Fermions are identified having indices of the Grassmann manifold. Conditions for integrability are analyzed and their physical meanings are discussed. We also address the nature of the exactly solvable part of the theory and find the infinitely many conserved quantities.

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1 Introduction

Understanding QCD, for example the quark confinement, is an important problem in theoretical physics which is not solved yet. One promising approach which receives much attention is the bosonization. t'Hooft found that QCD becomes equivalent to an effective field theory of mesons in the limit of a large number of colors (large-M limit).[1] Elaborating this idea, the Skyrme model was revised as an effective QCD theory where baryons are described by the solitons of the theory.[2][3] It was found that this scheme is very helpful in describing various properties of nucleons including their masses.[4] However the bridge between this phenomenological model and the physics of basic constituents is still missing. In this respect a lot of effort was devoted to two dimensional(2D) theory where a low-energy effective action can be derived directly from 2D-QCD using non-abelian bosonization.[5][6][7][8][9] Especillay in [5] it was shown that the effective low energy theory of the two dimensional QCD when bosonized is described by the Wess-Zumino-Witten(WZW) model with a mass term. This effective theory successfully describes the baryon spectrum and their various physical properties.[9] But the integrable structure of the theory is not known and the baryons are described by the low energy classical solutions and not by the solitons. As the integrable structure of the theory is important to understand the stability of baryons it is strongly desired to find a bosonization scheme with this structure.

In this paper we describe a new way of bosonization using the conformal embedding structure of conformal field theories that was introduced in [10][11]. This formalism leads to a bosonized theory of 2D-QCD which is integrable under proper limit. The integrable part of the resulting theory is a kind of nonabelian generalization of the sine-Gordon model[12] and it can be thought as a bosonization of a generalized massive Thirring model with a U(1) current-current interaction. This theory is described by an integrably deformed WZW model having Grassmann manifold as a symmetric space, see Ref.[12]. The deformation term corresponds to a fermion mass which only one flavor obtains. A similar analysis as in [12][13][14][15] can be applied to the present theory leading to a construction of the Lax pair, solitons and Backlund transformation. The theory corresponds to the bosonization of 2D-QCD when we remove the U(1) current interaction term. This interaction term which is related to the particle pair production, does not significantly change the integrability of the theory as it does not alter the degenerate structure of vaccum preserving the topological notion of solitons. We can make the contribution of this term negligible by taking proper limit. Main results of our analysis of 2D-QCD bosonization with the integrability condition are (related to 4-dimensional real world); 1)2D integrability under proper limit(baryons emerge as solitons in the theory of mesons), 2)only one fermion is massive in 2D theory(top quark is much massive than other quarks), 3)Grassmann manifold is descibed by $SU(M+N)/SU(M)\times SU(N)\times U(1)$ symmetric space (quarks have only two quantum numbers, i.e. flavor and color). Generalization of our method for bosonizing the massive GNO fermions corresponding to other symmetric spaces in Ref.[10] will be appeared elsewhere.[16]

2 GNO fermions and bosonization

The action of two dimensional QCD with M color and N flavor massive Dirac fermions is

$$S=\int d^2x[\sum_{a=1,i=1}^{M,N}\bar{\Psi}^{ai}\{-i\delta_a^b\ \partial\!\!\!/ - A\!\!\!/_\alpha(T^\alpha)_a^b\}\Psi_{bi}+\sum_{i=1}^{N}m_i^{(q)}\sum_{a=1}^{M}\bar{\Psi}^{ai}\Psi_{ai}-\frac{1}{2e_c^2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}], \quad (1)$$

where T^α are the generators of the color $SU(M)$ group. Note that we assign different fermion masses $m_i^{(q)}$ for each flavor. This theory has $U(MN)$ global symmetry when we neglect the mass term. The analysis of 2D-QCD through "non-abelian bosonization" introduced by Witten[17] have been achieved in [5][6][7][8][9]. These papers analyze 2D-QCD in the semi-classical limit and obtain the low-lying baryon spectrum[5][6], multibaryons[7] and matrix elements like various quark content.[8] Their treatment was in the spirit of Skyrme model[2] where baryons composed of M quarks can be treated as simple solitons in the bosonic language. The bosonized theory of 2D-QCD is the gauged WZW model with action on $U(MN)$ group manifold;

$$\begin{aligned}
S[f,A,\bar{A}] &= S_{WZW}[f] \\
&+ \frac{1}{2\pi}\int d^2x\mathrm{Tr}[iAf\bar{\partial}f^\dagger+i\bar{A}f^\dagger\partial f-Af\bar{A}f^\dagger+\bar{A}A] \\
&+ \sum\frac{m_i^2}{2\pi}N_\mu\int d^2x\mathrm{Tr}(f_{ai,ai}+f^\dagger_{ai,ai})-\frac{1}{2e_c^2}\int d^2x\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} \quad (2)
\end{aligned}$$

Here $S_{WZW}[f]$ is the action of the WZW theory with $f\subset U(MN)$. $\partial(\bar{\partial})$ denotes the derivative with respect to $z=x+t(\bar{z}=x-t)$. N_μ denotes normal ordering at mass μ and the bosonic mass m_i is related to $m_i^{(q)}$ and μ.[18] This theory, though quite interesting, does not permit the integrability analysis which is essential for Skyrme model approach.

Instead of above formalism we use in this paper a gauged WZW model with action on $G=SU(M)\times SU(N)\times U(1)$ group manifold. The idea is to express $f\subset U(MN)$ matrix elements in eq.(2) in terms of the group elements $g\subset G$ and makes the theory integrable. The key expression for our approach is

$$f_{ai,a'i'}=\frac{1}{2}\mathrm{Tr}g^{-1}p_{ai}^{(1)}g(p_{a'i'}^{(1)}-ip_{a'i'}^{(2)})=\frac{1}{2}\mathrm{Tr}g^{-1}p_{ai}^{(2)}g(p_{a'i'}^{(2)}+ip_{a'i'}^{(1)}). \quad (3)$$

In the following we explain the motivation of our approach and notations used in eq.(3). The WZW model on the $G=SU(M)\times SU(N)\times U(1)$ group which is obtained using eq.(2),(3) is conformally equivalent to the original WZW model on $F=U(MN)$ group, i.e. the difference of their two Virasoro algebras, $\mathcal{L}_{\mathcal{F}}-\mathcal{L}_{\mathcal{G}}$ has vanishing c-number. This fact is due to the underlying symmetric space structure in eq.(3). Ref.[11] indeed shows that all possible subalgebras of classical algebra which are conformally embedded to a larger group F can be found directly from the known classification of symmetric spaces. The symmetric space G'/G for our concern is the AIII type Grassmann manifold with $G'=SU(M+N)$.

Related with this conformal embedding structure is the theorem due to Goddard, Nahm and Olive[10] (GNO for short); a necessary and sufficient condition for the algebraic coincidence of Sugawara energy momentum tensor with that of the free fermions is there exists a group $G' \subset G$ such that G'/G is a symmetric space with the fermions transforming under G just as the tangent space to G'/G does. Based on this theorem they found all the fermionic theories for which an equivalent WZW bosonic action can be constructed. Bosonization of 2D-QCD exactly fits in this category with $F = U(MN)$, $G' = SU(M+N)$ and $G = SU(M) \times SU(N) \times U(1)$ groups. It is interesting to note that there is no symmetric space which has $G = SU(L) \times SU(M) \times SU(N) \times U(1)$ which means fermions at most have two quantum nembers, flavor and color. Let us express the group elements $g \subset G'$ which also belongs to the group G as $(M+N) \times (M+N)$ matrix;

$$g = \begin{pmatrix} g_N \subset SU(N) & 0 \\ 0 & g_M \subset SU(M) \end{pmatrix} e^{\theta T}, \tag{4}$$

with

$$T = \frac{-i}{M+N} \begin{pmatrix} MI_{N\times N} & 0 \\ 0 & -NI_{M\times M} \end{pmatrix}. \tag{5}$$

Type AIII space, called the hermitian symmetric space has the complex structure allowing following decomposition. Its **p** generators, the vector space complement of Lie algebra **g** of G in $\mathbf{g}'$ of G', i.e. $\mathbf{g}' = \mathbf{g} \oplus \mathbf{p}$, can be partitioned into MN families having two elements each; $p^{(1)}_{ai}$, $p^{(2)}_{ai}$($a = 1, M$; $i = 1, N$). Explicitly their matrix elements are $(p^{(1)}_{ai})_{lm} = \delta_{a+N,m}\delta_{i,l} + \delta_{a+N,l}\delta_{i,m}$($l, m =$ 1,M+N) and $(p^{(2)}_{ai})_{lm} = -i\delta_{a+N,m}\delta_{i,l} + i\delta_{a+N,l}\delta_{i,m}$. They satisfy following commutation relations

$$[T, p^{(1)}_{ai}] = p^{(2)}_{ai}, \quad [T, p^{(2)}_{ai}] = -p^{(1)}_{ai}. \tag{6}$$

Using properties $p^{\dagger}_{ai} = p_{ai}$, $\mathrm{Tr} p^{(k)}_{ai} p^{(k')}_{a'i'} = 2\delta_{aa'}\delta_{ii'}\delta_{kk'}$ we can easily show that

$$g^{-1} p^{(1)}_{ai} g = \frac{1}{2} f_{ai,a'i'}(p^{(1)}_{a'i'} + ip^{(2)}_{a'i'}) + \frac{1}{2} f^{*}_{ai,a'i'}(p^{(1)}_{a'i'} - ip^{(2)}_{a'i'}). \tag{7}$$

Then the unitarity of f is proved as

$$2\delta_{aa',ii'} = \mathrm{Tr}(g^{-1} p^{(1)}_{ai} g)(g^{-1} p^{(1)}_{a'i'} g) = 2(ff^{\dagger})_{aa',ii'}. \tag{8}$$

The equality of first and second expression for f in eq.(3) can be shown using the relation $e^{-\theta T} p^{(1)}_{ai} e^{\theta T} = \cos\theta p^{(1)}_{ai} - \sin\theta p^{(2)}_{ai}$ and the fact that $U(1)$ commutes with every elements of G.

We now express the bosonic currents $C^{m(M)}_{a'i',ai}(f^{-1}\partial f)_{ai,a'i'}$ and $C^{n(N)}_{a'i',ai}(f^{-1}\partial f)_{ai,a'i'}$ in terms of the group G bosonic currents using eq.(3). Here $C^{m(M)}_{a'i',ai}$ and $C^{n(N)}_{a'i',ai}$ are defined as

$$[T^{(M)}_m, p^{(1)}_{ai} - ip^{(2)}_{ai}] = i(p^{(1)}_{a'i'} - ip^{(2)}_{a'i'})C^{m(M)}_{a'i',ai}, \;\; [T^{(N)}_n, p^{(1)}_{ai} - ip^{(2)}_{ai}] = i(p^{(1)}_{a'i'} - ip^{(2)}_{a'i'})C^{n(N)}_{a'i',ai}, \tag{9}$$

where $T_m^{(M)}(m = 1, M^2-1), T_n^{(N)}(n = 1, N^2-1)$ are generators of Lie algebra $\mathbf{g'} = \mathbf{su(m+n)}$ corresponding to the subalgebra $\mathbf{su(m)}$ and $\mathbf{su(n)}$ each. They are normalized as $\mathrm{Tr}T_m^{(M)}T_{m'}^{(M)} = 2\delta_{mm'}$, $\mathrm{Tr}T_n^{(N)}T_{n'}^{(N)} = 2\delta_{nn'}$ and $\mathrm{Tr}T_m^{(M)}T_n^{(N)} = 0$. Explicitly $C_{a'i',ai}^{m(M)} = -i\delta_{ii'}(T_m^{(M)})_{a'+N,a+N}$ and $C_{a'i',ai}^{n(N)} = i\delta_{aa'}(T_n^{(N)})_{i,i'}$ and they have following properties

$$[p_{ai}^{(1)} - ip_{ai}^{(2)}, p_{a'i'}^{(1)} + ip_{a'i'}^{(2)}] = -2iC_{ai,a'i'}^{m(M)}T_m^{(M)} - 2iC_{ai,a'i'}^{n(N)}T_n^{(N)} + 4i\frac{M+N}{MN}\delta_{aa'}\delta_{ii'}T \tag{10}$$

and

$$C_{ai,a'i'}^{m(M)}C_{a'i',ai}^{m'(M)} = -2N\delta_{mm'}, \quad C_{ai,a'i'}^{n(N)}C_{a'i',ai}^{n'(N)} = -2M\delta_{nn'}, \quad C_{ai,a'i'}^{m(M)}C_{a'i',ai}^{n(N)} = 0. \tag{11}$$

Using all these properties the $SU(M)$ current $C_{a'i',ai}^{m(M)}(f^{-1}\partial f)_{ai,a'i'}$ becomes

$$\begin{aligned} & \frac{1}{2}C_{a'i',ai}^{m(M)}(f^{-1})_{ai,a''i''}\mathrm{Tr}g^{-1}p_{a''i''}^{(1)}g[g^{-1}\partial g, (p_{a'i'}^{(1)} - ip_{a'i'}^{(2)})] \\ = \; & iC_{a'i',ai}^{m(M)}\delta_{aa''}\delta_{ii''}C_{a''i'',a'i'}^{m'(M)}(g^{-1}\partial g)_{m'}^{(M)} = -2iN(g^{-1}\partial g)_m^{(M)} \end{aligned} \tag{12}$$

where $g^{-1}\partial g = (g^{-1}\partial g)_m^{(M)}T_m^{(M)} + (g^{-1}\partial g)_n^{(N)}T_n^{(N)} + \partial\theta T$. Similarly the $SU(N)$ current is $C_{a'i',ai}^{n(N)}(f^{-1}\partial f)_{ai,a'i'} = -2iM(g^{-1}\partial g)_n^{(N)}$ and $U(1)$ current is $(f^{-1}\partial f)_{ai,ai} = iMN\partial\theta$. These expressions show that the currents satisfy level-N $SU(M)$ and level-M $SU(N)$ Kac-Moody algebras each, which can be understood as the bosonic correspondent of GNO result.[10]

The action of gauged WZW theory, eq.(2), can also be expressed in terms of the elements of group G. For example, the kinetic term is calculated to be

$$\begin{aligned} & (f^{-1}\partial f)_{ai,a'i'}(f^{-1}\bar{\partial} f)_{a'i',ai} \\ = \; & 2N(g^{-1}\partial g)_m^{(M)}(g^{-1}\bar{\partial} g)_m^{(M)} + 2M(g^{-1}\partial g)_n^{(N)}(g^{-1}\bar{\partial} g)_n^{(N)} - MN\partial\theta\bar{\partial}\theta \\ = \; & M\mathrm{Tr}g_1^{-1}\partial g_1 g_1^{-1}\bar{\partial} g_1 + N\mathrm{Tr}g_2^{-1}\partial g_2 g_2^{-1}\bar{\partial} g_2 - MN\partial\theta\bar{\partial}\theta, \end{aligned} \tag{13}$$

where $g_1(g_2)$ means the group element g in eq.(4) with $g_M = e^{\theta T} = 1 (g_N = e^{\theta T} = 1)$. When we take the gauge field $A_{ai,a'i'} = A_m(T_m^{(M)})_{a+N,a'+N}\delta_{ii'}$, terms containing gauge field are also expressed by $A = A_m T_m^{(M)}$ and g. In this way the gauged WZW action, eq.(2), of $MN \times MN$ matrix f can be rewritten as an action of $(M+N)\times(M+N)$ matrix g;

$$\begin{aligned} & MS_{WZW}[g_1] + NS_{WZW}[g_2] - \frac{1}{2e_c^2}F_{\mu\nu}F^{\mu\nu} + \frac{m_i^2}{2\pi}N_\mu \mathrm{Tr}(g_1g_2e^{\theta T})^{-1}p_{ai}^{(1)}(g_1g_2e^{\theta T})p_{ai}^{(1)} \\ + \; & \int d^2x[-\frac{MN}{8\pi}\partial\theta\bar{\partial}\theta + \frac{N}{2\pi}\mathrm{Tr}(iAg_2\bar{\partial}g_2^{-1} + i\bar{A}g_2^{-1}\partial g_2 - Ag_2\bar{A}g_2^{-1} + A\bar{A})]. \end{aligned} \tag{14}$$

To treat the gauge field we follow the prescription of Ref.[9]. First take the gauge $\bar{A} = 0$ and integrate out the A field to obtain an action having $-(\frac{e_c N}{4\pi})^2 \int d^2x \mathrm{Tr}H^2$. Here H is defined by $\bar{\partial}H = ig_2\bar{\partial}g_2^{-1}$ with the boundary condition $H(-\infty, \bar{z}) = 0$. In the strong coupling limit $e_c/m_i \to \infty$ the fields g_2 which contribute to H will become infinitely heavy and can be ignored. We then get the low-energy effective action with a substitution $g_2 = 1$ and a

suitable identification of the mass m_i. To make an integrable theory from the QCD theory in eq.(14), we take two modifications on the theory together with $g_2 = 1$; take the mass $m_i = 0$ except $i = 1$ and add an interaction term of Thirring type $-\frac{\pi}{2M(N+1)}\bar{\Psi}_{ai}\gamma_+\Psi_{ai}\bar{\Psi}_{a'i'}\gamma_-\Psi_{a'i'}$. The action corresponding to the integrable part of the effective theory, which we will explain below, is

$$S = MS[l] + M\frac{m^2}{2\pi}\int d^2x \mathrm{Tr} l^{-1}plp \tag{15}$$

where m is related to the quark mass m_1.[18] Here l,p are $(N+1)\times(N+1)$ matrices

$$\begin{aligned} l &= \begin{pmatrix} g_N \subset SU(N) & 0 \\ & \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\phi} & \cdots & 0 \\ 0 & e^{i\phi} & \\ & \cdots & \\ & 0 & e^{-iN\phi} \end{pmatrix} \\ p &= \begin{pmatrix} 0 & 0 & \cdots & 1 \\ & & & \\ 0 & 0 & \cdots & 0 \\ & \cdots & & \\ 1 & 0 & \cdots & 0 \end{pmatrix}, \end{aligned} \tag{16}$$

with $\phi = \frac{-1}{N+1}\theta$. The assumption that only $i = 1$ fermions are massive is natural when we consider the fact that top quark is much massive than other quarks in real world. As the $U(1)$ current is bosonized as $J = \bar{\Psi}_{ai}\gamma_+\Psi_{ai} \rightarrow i\frac{MN}{2\pi}\partial\theta$, U(1) current interaction is bosonized to be $\frac{1}{8\pi}\frac{MN^2}{N+1}\partial\theta\bar{\partial}\theta$ and shifts the coefficent of the kinetic term for θ in the action. This gives the kinetic term for ϕ in $\frac{M}{8\pi}\mathrm{Tr} l^{-1}\partial l l^{-1}\bar{\partial} l$ of eq.(15) to be $-\frac{1}{8\pi}\frac{MN}{N+1}\partial\theta\bar{\partial}\theta$. As this term does not contribute to the potential in bosonic picture, the soliton configuration originated from the degeneracy of vacuum is maintained. In fermionic picture the current interaction term contributes to scattering process between solitons by creating or annihilating fermion-antifermion pairs such that they dress the solitons. Note the current interaction becomes negligible when we take large-N or large-M limit in the fermionic picture, while the relative contribution to kinetic term remains finite in bosonic picture. The second term in the action is originated from the mass term and is explicitely $e^{-i(N+1)\phi}(g_N)^{-1}_{11} + e^{i(N+1)\phi}(g_N)_{11}$ which coincide with the result from eq.(14). So 2D-QCD with a massive fermion is bosonized to be an integrable theory deformed by the U(1) current interaction term.

3 Integrability, solitons and conservation laws

The action in eq.(15), especially the mass term bilinear in l, permits the equation of motion to be expressed in a zero curvature form, leading to the integrable theory.[12][13] To see this, we first apply the variation on l to obtain the equation of motion;

$$\{-\bar{\partial}(l^{-1}\partial l) - m^2[p, l^{-1}pl]\}l^{-1}\delta l = 0. \tag{17}$$

Using the simple relation

$$\partial(l^{-1}pl) + [l^{-1}\partial l, l^{-1}pl] = 0 \tag{18}$$

eq.(17) can be recasted in a zero curvature form;

$$[\partial + l^{-1}\partial l + \lambda p, \bar{\partial} - \frac{m^2}{\lambda} l^{-1}pl] = 0. \tag{19}$$

The integrability of the theory introduces the following Backlund transformation[BT][15];

$$\begin{aligned} l^{-1}\partial l - \tilde{l}^{-1}\partial\tilde{l} + m^2\eta[l^{-1}p\tilde{l}, p] &= 0 \\ \eta\bar{\partial}(l^{-1}p\tilde{l}) + l^{-1}pl - \tilde{l}^{-1}p\tilde{l} &= 0 \end{aligned} \tag{20}$$

where η is a BT parameter characterizing the solitons. The Backlund transformation offers us the ability to calculate the 1-soliton solution of the theory starting from the vacuum solution, for which $\tilde{l} = 1$, and multi-soliton solutions using non-abelian superposition rules[15]. When we parametrize l for 1-soliton as

$$l = \begin{pmatrix} e^{i\varphi} & & & & \\ & 1 & & & \\ & & \cdots & & \\ & & & 1 & \\ & & & & e^{-i\varphi} \end{pmatrix}, \tag{21}$$

the BT becomes

$$\begin{aligned} \partial\varphi - 2m^2\eta\sin\varphi = 0 \\ \eta\bar{\partial}\varphi - 2\sin\varphi = 0. \end{aligned} \tag{22}$$

The one soliton solution that can be obtained from eq.(22) is the well-known sine-Gordon soliton which in the static case is[14]

$$\varphi(x) = 2\tan^{-1} e^{4m(x-x_0)}. \tag{23}$$

This solution interpolates two different vacuua of the theory, i.e. $\varphi(x \to -\infty) = 0$, $\varphi(x \to \infty) = \pi$, which are given by the minimum of potential $-\frac{Mm^2}{2\pi}\mathrm{Tr} l^{-1}plp = -\frac{Mm^2}{\pi}\cos 2\varphi$, i.e. $\varphi = n\pi$.

Another important aspect of the integrable theory is it has infinitely many conserved quantities.[13] The interpretation of the zero curvature equation (19) as a compatibility condition of two linearized equations permits us to write the conserved currents in an iterative form.[13] Let us decompose the $(N+1)\times(N+1)$ matrix $l^{-1}\partial l$ and $l^{-1}pl$ as following;

$$l^{-1}\partial l = \begin{pmatrix} a+d & c & 0 \\ -c^\dagger & e & 0 \\ 0 & 0 & a-d \end{pmatrix}, \quad l^{-1}pl = \begin{pmatrix} 0 & 0 & r-is \\ 0 & 0 & q^\dagger \\ r+is & q & 0 \end{pmatrix} \tag{24}$$

where c, q are $1 \times (N-1)$ matrices while e is an $(N-1) \times (N-1)$ matrix with property $e = -e^\dagger$. a, d are pure imaginary numbers while r, s are real. Then the conserved currents are calculated iteratively with these matrix elements by following formulae;

$$\begin{aligned}
\alpha_i &= \frac{-1}{m^2}\{(\partial - a + e)\alpha_{i-1} - \frac{1}{2}c^\dagger(\psi_{i-1} + \beta_{i-1})\} \\
\beta_i &= \frac{-1}{2m^2}(\partial\beta_{i-1} + c\alpha_{i-1} + d\psi_{i-1}) \\
J_i = \partial\psi_i &= \frac{1}{m^2}\{(c\partial - ac + ce + \frac{1}{2}dc)\alpha_{i-1} + \frac{1}{2}(d\partial - cc^\dagger)\beta_{i-1} - \frac{1}{2}(cc^\dagger - d^2)\psi_{i-1}\} \\
\bar{J}_i = \bar{\partial}\psi_i &= -q\alpha_{i-1} - is\beta_{i-1} - r\psi_{i-1}.
\end{aligned} \tag{25}$$

Now stating with $\alpha_0 = \beta_0 = 0, \psi_0 = 1$, all higher currents satisfying $\bar{\partial}J_i = \partial\bar{J}_i$ can be calculated. For i=1, the conservation law becomes

$$\frac{1}{2m^2}\bar{\partial}(cc^\dagger - d^2) = \bar{\partial}r. \tag{26}$$

This is just the conservation of energy. Higher order conserved currents in general become non-local. For example,

$$\begin{aligned}
J_2 &= \frac{1}{2m^4}\{c(\partial - a + e + \frac{d}{2})c^\dagger - \frac{1}{2}(d\partial - cc^\dagger)d\} + J_1\psi_1 \\
\bar{J}_2 &= \frac{-1}{2m^2}(qc^\dagger - isd)\} + \bar{J}_1\psi_1.
\end{aligned} \tag{27}$$

This however can be made local if we subtract non-local terms containing ψ_1 from currents. Note that the non-local terms themselves satisfy the conservation law, $\bar{\partial}(J_1\psi_1) = \partial(\bar{J}_1\psi_1)$. Similarly the third-order currents after subtracting non-local terms containing ψ_1, ψ_2 are

$$\begin{aligned}
J_3 &= \frac{-1}{2m^6}c(\partial - a + e + \frac{d}{2})^2c^\dagger - \frac{1}{4m^6}(cc^\dagger - \frac{d^2}{2})(cc^\dagger - d^2) + \frac{1}{8m^6}(d\partial - cc^\dagger)(\partial d - cc^\dagger) \\
\bar{J}_3 &= \frac{1}{2m^4}q(\partial - a + e + \frac{1}{2}d)c^\dagger - \frac{1}{4m^4}is(\partial d - cc^\dagger).
\end{aligned} \tag{28}$$

These conservation laws can be directly checked using the equation of motions (17),(18) which can be expressed in a new form as

$$\begin{aligned}
&\bar{\partial}a = \bar{\partial}e = 0, \quad \bar{\partial}c + m^2q = 0, \quad \bar{\partial}d + 2ims = 0 \\
&\partial q + (a-d)q - (r+is)c - qe = 0 \\
&\partial r - 2ids + \frac{1}{2}(cp^\dagger + pc^\dagger) = 0 \\
&\partial s + 2idr + \frac{i}{2}(cp^\dagger - pc^\dagger) = 0.
\end{aligned} \tag{29}$$

These infinitely many conservation laws guarantee the shape-preserving property of solitons after collisions and will be helpful in fixing the scattering amplitudes.[19] 2D-QCD is an almost integrable system and the difference from the exactly integrable system is the change of the coefficient of U(1) kinetic term. We speculate the baryons after collision can have zero-soliton quantum fluctuations around them, making a slight deformation of solitons.

4 Discussions

Finally we point out the different nature of quantum fluctuation around solitons of present theory from the conventional one.[9] Usually the quantum fluctuation is described by the matrix $l(x,t) = A(t)l_0(x)A^{-1}(t)$ [5] where

$$A = \begin{pmatrix} z_1 & y_2 & \cdots & y_N & 0 \\ z_2 & & & & 0 \\ & & a_{ij} & & \\ z_N & & & & \\ 0 & & 0 & & \zeta \end{pmatrix}. \tag{30}$$

This form describes finite-energy configuation around soliton when the soliton has the property $l_0(x) \to 1$ as $x \to \pm\infty$. In the present theory $l_0(x) \not\to 1$ when $x \to \infty$, and $l(x,t)$ does not correspond to finite-energy configuration. Indeed the mass term is

$$\mathrm{Tr} l^{-1} p l p - \mathrm{Tr} l_0^{-1} p l_0 p = 2(\cos 2\varphi - \cos\varphi)(z_1 z_1^* - 1). \tag{31}$$

As the soliton $\varphi \to \pi$ when $x \to \infty$, the energy of new configuration becomes infinite except the case $|z_1| \to 1$ as $x \to \infty$. Similarly the term $[A^{-1}\dot{A}, l_0][A^{-1}\dot{A}, l_0^\dagger]$ contained in $S_0[Al_0A^{-1}] - S_0[l_0]$ becomes

$$4(1 - \cos\varphi)\{\dot{z}_i^* \dot{z}_i + \dot{\zeta}^* \dot{\zeta} + (z_i^* \dot{z}_i)^2 + (\zeta^* \dot{\zeta})^2\}, \tag{32}$$

which also becomes infinite except the case $|z_1| \to 1$. But when we take $|z_1| = 1$ without dependence on x, it only results $l(x,t) = l_0(x)$. So x-dependence of z_1 and A is necessary to make a finite-energy configuration, resulting a pulsating solution. The detailed study of this configuration is not performed yet and we defer it for future analysis.

Recently there appears extensive development called the decoupled bosonization.[19]-[21] It bosonize 2D-QCD by rewriting the theory in terms of gauge invariant fields and describes massless fermions in terms of positive and negative level WZW models, ghosts and massive bosonic excitations. Like our formalism it also has led to interesting insights into the characterestics of the model, such as its integrability, degeneracy of the vacuum, and higher symmetry algebras. And its integrability condition is valid for the quantum theory as well. But this model displays a complicated set of constraints and the expression for the integrability is non-local. So it seems difficult to find explicitly the classical soliton solutions and to analyze the mesonic spectrum and their physical properties using the method of Skyrme model. In addition the integrability condition does not survive for massive case in this approach.[20] It can be thought that our formulation, specially the method treating the mass term, could find interesting application in the decoupled formulation. One interesting common feature shared by two formulations is the so-called quasi-integrability, that is particle pair production is not entirely suppressed.[21] It seems worthwhile to make a detailed analysis

of two formulations simultaneously and make a simple and exact bosonization formulation taking merit of each formalism.

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